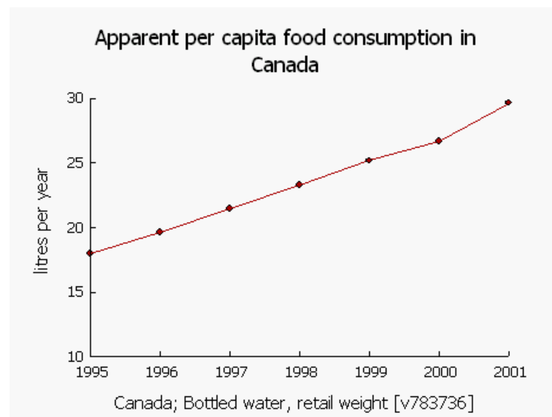
Unit 1

1.1 Order of operations

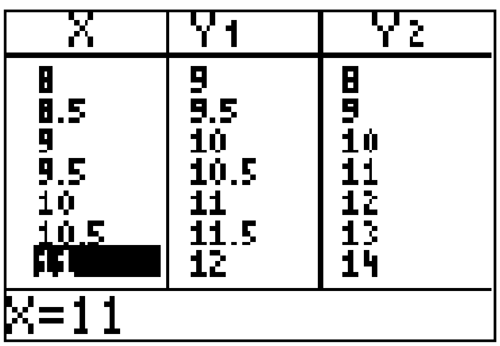
1.2 Linear equations and inequalities

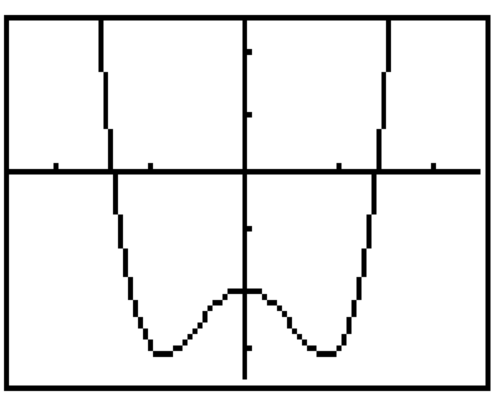
1.3 Graphs and intercepts

1.4 Tables, graphs, and equations



1.5 Functions





P ARENTHESES
E XPONENTS
M ULTIPLY
D IVIDE
A DD
S UBTRACT

1.1 Order of operations and Scientific notation

The **order of operations** specifies the order that is followed to simplify expressions.

1. Grouping: Simplify within grouping symbols (such as parentheses, brackets, absolute value signs, square root symbols). Start with the innermost symbols and work outward.
2. Exponents: Evaluate all exponents and radicals.
3. Multiply and divide: All multiplication and division should be done in order from left to right.
4. Add and subtract: All addition and subtraction should be done in order from left to right.

Note: Fraction bars are also considered a grouping symbol. Operations in the numerator and the denominator should be done before the fraction is simplified.

Scientific notation is a notation that is used to write very large or very small numbers in a compact form. The notation looks like A x 10^n where A is an integer such that $1 \leq |A| < 10$. An easy way to find the exponent, n, is to count the number of decimal places the decimal is moved in order to have an appropriate value of A. A positive value of n will represent a number bigger than 1 and a negative value of n will represent a number smaller than 1. For example, 1,234,000,000 is written as 1.234 x 10^9. The number 0.00045 is written as 4.5 x 10^{-4}. The graphing calculator notation looks like A E n where A and n are the same as above. The number 3780 is written in scientific notation as 3.78 x10^3 and will appear on the calculator as 3.78 E 3.

Homework:

Simplify the following:

1. $2 + 3(7 - 1)$
2. $\sqrt{81} + 5(3 - 9)$
3. $5[3 + 2(6+3)]$
4. $\sqrt[3]{-125}$
5. $64 \div 16 \cdot 4 + 2$
6. $-\sqrt[4]{81}$
7. $5[-2 + -3(4 - 9)] \div 2$
8. -6^2
9. $\frac{431-970}{320-112}$
10. $\sqrt{\frac{169}{9}}$
11. $(-3)^4$
12. $\sqrt{6^2 - 4 \cdot 3 \cdot -2}$
13. $\frac{4(5-8)}{3} - \frac{8}{-4}$
14. $\frac{-7+\sqrt{7^2-4(3)(2)}}{2(3)}$
15. $\frac{2^3-3+1}{-2} + \frac{(-4)^2}{8}$
16. $2|-7 - 11| + 4$
17. $|6 - 9| + 4(3 - 7)^2$
18. $\sqrt{64 - 25} + 2(-5)$

Rewrite the following in scientific notation:

19. 470,000

20. 1,512,000,000

21. 0.00000341

22. 0.0006

Rewrite the following in standard form:

23. 1.54×10^{-5}

24. 3.9×10^{4}

25. 4 E 3

26. -2.1 E -6

27. Compute with the aid of your calculator. Write your answer in both standard and scientific notation.

$$\frac{\left(4.1\times10^{4}\right)\left(2.5\times10^{2}\right)}{5\times10^{-3}}$$

28. Lake Eola (which is actually a sinkhole) has an area of approximately 1,001,880 square feet and is an average of 55 feet deep.

A. Find the approximate volume of Lake Eola in cubic feet.

B. If 1 cubic foot of water is equivalent to 7.48 gallons, how many gallons of water are in Lake Eola? Write your answer in standard form and in scientific notation.

29. In July 2012, the public debt was approximately $11,122,280,000,000.

A. Write this number in scientific notation.

B. The population of the United States in July 2012 was approximately 314 million. What was the debt per person (per capita debt) in July 2012?

1.2 Linear Equations and Inequalities

An **equation** is a mathematical statement that two expressions are equal. Linear equations are first-degree equations (the variable has an exponent of one). To solve linear equations, isolate the variable on one side of the equal sign. Recall that you can add or subtract the same number to both sides of an equation or you can multiply or divide the same number (not zero) to both sides of an equation. Equations may be solved algebraically, numerically, and graphically.

Steps to solve algebraically:

1. Simplify each side of the equation separately. Apply grouping symbols and collect like terms.
2. Move all terms containing variables to one side of the equation and constants to the other by appropriately adding or subtracting terms to both sides of the equation.
3. Divide both sides of the equation by the coefficient of the variable.

Examples:

1. $4(2x - 3) = 12$

$8x - 12 = 12$	Distribute to remove the parentheses.
$8x - 12 + 12 = 12 + 12$	Add 12 to isolate the term with x.
$\frac{8x}{8} = \frac{24}{8}$	Divide by 8 to solve for x.
$x = 3$	

2. $3.7x + 23.4 = 1.2x - 10.4$

$3.7x + 23.4 - 1.2x = 1.2x - 1.2x - 10.4$	Subtract 1.2x from both sides.
$2.5x + 23.4 = -10.4$	Simplify.
$2.5x + 23.4 - 23.4 = -10.4 - 23.4$	Subtract 23.4 from both sides.
$\frac{2.5x}{2.5} = \frac{-33.8}{2.5}$	Divide both sides by 2.5.
$x = -13.52$	

To solve an equation numerically, we use a table of values.

Steps to solve numerically:

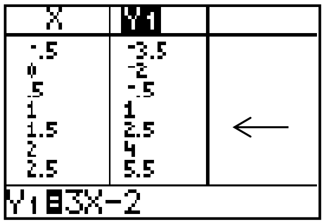
1. Make a table of values which satisfy the expressions on both sides of the equal sign for values of the variable.

2. Determine where the two expressions are equal for the same value of the variable.

Note: If only one expression has a variable, then you can just make a table for that side of the equation and find where the output is equal to the constant on the other side.

Examples:

1. Use the table to solve the equation. 3x - 2 = 2.5 Indicate on the table how you found your solution.



X	Y1	
-.5	-3.5	
0	-2	
.5	-.5	
1	1	
1.5	2.5	←
2	4	
2.5	5.5	

Y_1=3X-2

Look for 2.5 in the output (Y_1) column, the x-value is the solution.

x = <u> 1.5 </u>

2. Solve 2x – 8 = -4 numerically.

   Make a table of values for 2x – 8:

x	-1	0	1	**2**	3
2x-8	-10	-8	-6	**-4**	-2

   Look at the output values to find where they are equal to -4. The solution is x = 2.

3. Solve 3x + 8 = 2x + 7

   Make a table of values for both sides of the equation:

x	-2	**-1**	0	1	2	3	4
3x + 8	2	**5**	8	11	14	17	20
2x + 7	3	**5**	7	9	11	13	15

   Look at the output values to find where they are equal for the same value of x. The solution is x = -1.

To solve graphically, we use graphs. For now, we will be given the graphs of both sides of the equation. Later, we will construct our own graphs to solve graphically.

Steps to solve graphically:

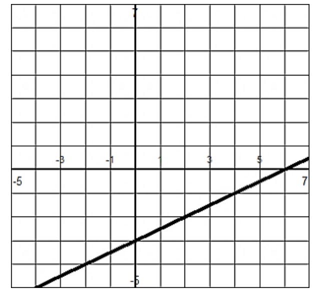
1. A graph should be made for both sides of the expression by setting y equal to each side and drawing the appropriate graph.

2. Determine where the two graphs are equal for the same value of the variable. This will be the intersection point of the two graphs.

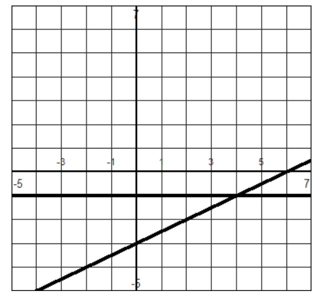
Note: If only one expression has a variable, then you can just make a graph for that side of the equation and find where the output is equal to the constant on the other side.

Examples:

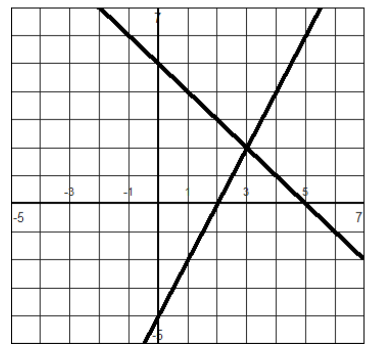
1. Use the graph of y = ½ x – 3 to solve the equation ½ x – 3 = -1.



We are looking for the x-value of a point which has an output value of -1. To find this, draw a horizontal line across the graph at y = -1.



Find the point of intersection and identify the x-value of the intersection point. We can see that the intersection occurs at x = 4 which will be the solution to the equation.



2. Use the graph shown to solve the equation 2x – 4 = -x + 5. The lines y = 2x – 4 and y = -x + 5 have been drawn.

   The solution to the equation is the x-value of the intersection point. These lines intersect at the point (3, 2) so the solution to the equation is x = 3.

A **formula** contains more than one variable in an equation. Formulas can be solved for a chosen variable using the same steps as equations but the chosen variable must be isolated on one side of the equation with all other variables and constants on the other.

Examples:

1. Solve 3x + 5y = 7 for y.

$3x + 5y - 3x = 7 - 3x$	Subtract 3x from both sides to isolate the term with y.
$\frac{5y}{5} = \frac{7-3x}{5}$	Divide both sides by 5 to solve for y.
$y = \frac{7}{5} - \frac{3}{5}x$	

2. Solve P = 2L + 2W for L.

$P - 2W = 2L + 2W - 2W$	Subtract 2W from both sides to isolate the term with L.
$\frac{P - 2W}{2} = \frac{2L}{2}$	Divide both sides by 2 to solve for L.
$\frac{P}{2} - W = L$	

Interval notation is a notation used to write the solutions to inequalities without using inequality symbols. An interval is a set that consists of all real numbers between the numbers a and b. Brackets are used to indicate a closed interval or one that includes the endpoints. Parentheses are used to indicate an open interval or one that excludes the endpoints. To indicate an infinite interval, we use the infinity symbol, ∞, and parentheses are always used to indicate that infinity is open-ended. Interval notation should have the smallest solution on the left and largest solution on the right of the notation.

Examples:

1. $-1 \le x \le 4$ is written [-1, 4]
2. $x > 9$ is written $(9, \infty)$
3. $5 < x \le 8$ is written (5, 8]
4. $x \le 3$ is written $(-\infty, 3]$

Linear inequalities consist of two expressions related by an inequality symbol. Possible inequality symbols are < (less than), > (greater than), ≤ (less than or equal to), and ≥ (greater than or equal to). Linear inequalities are solved in a manner similar to equations except that if both sides of the inequality are multiplied or divided by a negative number, the inequality symbol must be reversed.

Example:

$-2x + 5 > -9$

$-2x + 5 - 5 > -9 - 5$ Subtract 5 to isolate the term with x.

$\frac{-2x}{-2} > \frac{-14}{-2}$ Divide by -2 to solve for x but must reverse the inequality symbol.

$x < 7$ In interval notation the solution is $(-\infty, 7)$.

Linear inequalities can also be solved numerically. The solution will be a set of real numbers. Find the input value where the inequality is equal and then determine which set of numbers (less or greater than that number) solve the inequality.

Examples:

1. Use the table to solve the inequality. $0.5(3x - 1) < 4$ Indicate on the table how you found your solution.

X	Y1	
-1	-2	
0	-.5	
1	1	
2	2.5	
3	4	
4	5.5	
5	7	

Y1=.5(3X-1)

Determine where the output is equal to 4. The expression is equal to 4 for an x-value of 3. The outputs are less than 4 for values of x which are less than 3, so the solution is $x < 3$ or $(-\infty, 3)$.

2. Use the table to solve the inequality. x + 1 ≤ 2(x – 4) (Note: Y_1 = x +1 and Y_2 = 2(x – 4))

X	Y_1	Y_2
8	9	8
8.5	9.5	9
9	10	10
9.5	10.5	11
10	11	12
10.5	11.5	13
11	12	14

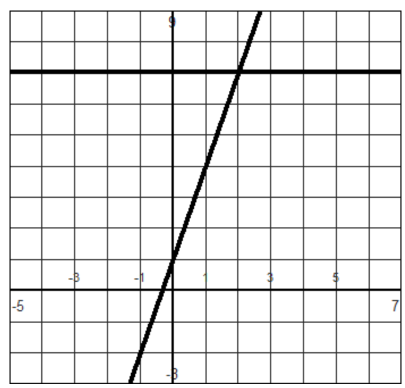
X=11

The outputs are equal for an x-value of 9. For input values less than 9, Y_1 is greater than Y_2 while for input values greater than 9, Y_1 is less than Y_2. The inequality is x + 1 ≤ 2(x – 4), therefore the solution is x ≥ 9 or [9, ∞).

Linear inequalities can also be solved graphically. The solution will again be a set of real numbers. Find the point of intersection and identify the input value of the intersection point. Next, greater than represents above when looking at a graph and less than is below. Identify the solutions to the inequality as the interval of values which correspond to the portion of the graph which satisfies the inequality.

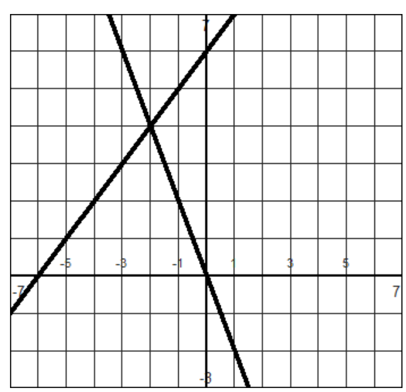
Examples:

1. Solve the inequality 3x + 1 ≥ 7 using the graph shown.



The graph shows the lines y = 3x + 1 and y =7. The point of intersection occurs at an input value of x = 2. We need to identify where the graph of y = 3x + 1 is above (greater than) the line y = 7. This occurs for all values to the right of x = 2. Therefore, the solution to the inequality is x ≥ 2 or the interval [2, ∞).

2. Solve the inequality -2x < x + 6 graphically given the graph shown.



The graph shows the lines y = -2x and y = x + 6. The graphs intersect at an input value of x = -2. Now, we need to identify where the graph of y = -2x is below (less than) the graph of y = x + 6. This occurs for values greater than x = -2. The solution to the inequality will be (-2, ∞).

Compound inequalities involve two inequality symbols. When solving, isolate the variable in the center and be sure to perform all operations on all three parts of the inequality.

Example:

$4 < 3x + 7 < 22$

$4 - 7 < 3x + 7 - 7 < 22 - 7$ Subtract 7 from all three parts to isolate the term with the variable.

$\frac{-3}{3} < \frac{3x}{3} < \frac{15}{3}$ Divide by 3 to solve for x.

$-1 < x < 5$ Solution in interval notation is (-1, 5).

<u>Homework:</u>

Solve the following equations algebraically and write the solutions in interval notation.

1. $4x + 7 = 19$
2. $-3x - 11 = 2$
3. $2x + 6 = 4x - 11$
4. $1.2x - 20 = x - 4$
5. $2(x + 3) - 5(2x + 1) = 4$
6. $\frac{2}{5}x - \frac{3}{4} = \frac{1}{10}$
7. $\frac{4}{3}\left(3x - \frac{1}{2}\right) - 5 = \frac{2}{3}x - 1$
8. $-1.5(3x + 2) - (x + 4.6) = 3.2$
9. $5y - 3(y + 3) = 2y + 7$
10. $5t + 6(3t + 2) = 4(t - 9)$
11. $1 - (4x - 7) = 2(2x + 5)$
12. $5(x + 3) + 2(1 - x) = 3x + 17$
13. A driver is 250 miles from home, driving towards home at an average speed of 55 miles per hour.
    A. Make a table that shows the driver's distance from home after 0, 1, 2, and 3 hours.
    B. Write a formula that calculates the distance D that the driver is from home after h hours.
    C. Use your formula to determine D when h = 3 hours. Does your answer agree with the value found in your table?
14. The amount of gasoline (in gallons) left in the tank of a car after traveling m miles can be approximated by the formula $G = 16 - \frac{1}{28}m$.
    A. How much gasoline is left in the tank of the car after traveling 130 miles?
    B. How many miles can the car travel before there is 2 gallons of gasoline left in the tank?

15. The number of people in a small town x years after 1960 can be estimated by the formula

P = 375 + 15x.

A. How many people are in the town in 1980?

B. In what year will the town have 1000 people?

Solve each equation numerically. Evaluate the expression for each value of x in the table. Then use the table to solve the equation.

16. Use the table to solve the equation. 4(x - 2) + 2(x + 3) = -8
Indicate on the table how you found your solution.

X	Y1	
-2	-14	
-1	-8	
0	-2	
1	4	
2	10	
3	16	
4	22	

Y1=4(X-2)+2(X+3)

17. 2x + 5 = 13

x	-2	0	2	4	6
2x + 5					

18. 2 – 3(5x + 4) = 20

x	-3	-2	-1	0	1
2 - 3(5x + 4)					

19. 7 – 3x = x + 11

x	-2	-1	0	1	2
7 - 3x					
x + 11					

20. $\frac{1}{4}(x+1) = \frac{1}{2}(2x+5) - 6$

x	-1	1	3	5	7
$\frac{1}{4}(x+1)$					
$\frac{1}{2}(2x+5) - 6$					

21. $0.8x + 1.2(2x – 3) = -3.6$

x	-2	-1	0	1	2
$0.8x + 1.2(2x – 3)$					

Solve the following formulas for the indicated variable:

22. $6x – 3y = 15$ for y

23. $A = 2w^2 + 4lw$ for *l*

24. $A = P(1 + rt)$ for t

Solve the following inequalities algebraically. Write the solutions in inequality and interval notation.

25. $5x – 9 > 11$

26. $-2x + 7 \leq -5$

27. $3x + 2 < 6x + 8$

28. $2(4 – x) \geq x + 3(x + 1)$

29. $-5 < 2x – 3 \leq 11$

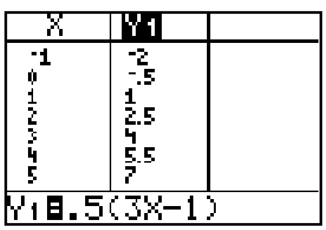
30. $4 < -3(2x + 5) < 12$

31. $\frac{x-13}{4} > -2$

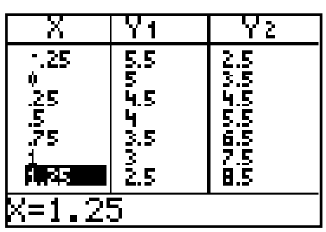
32. $\frac{-2x+3}{2} \leq 4$

Solve the inequalities numerically:

33. Use the table to solve the inequality. $0.5(3x - 1) < 4$ Indicate on the table how you found your solution.



34. Use the table to solve the inequality. $5 - 2x \geq 4x + 3.5$. Indicate on the table how you found your solution.



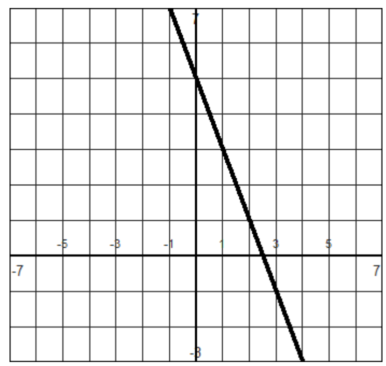
35. Make a table to solve $\frac{3}{4}x+1<-\frac{1}{4}x+3$. Be sure to indicate your solution.

x					
$\frac{3}{4}x+1$					
$-\frac{1}{4}x+3$					

36. Write -2 < x $\leq$ 4 in interval notation and graph the set on a number line.

Solve the following equations and inequalities graphically.

37. Given the graph of y = -2x + 5 as shown, solve:



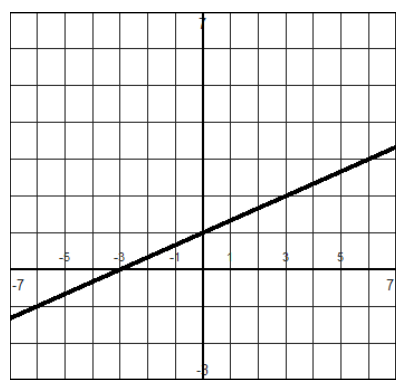
A. -2x + 5 = 0

B. -2x + 5 = 3

C. -2x + 5 < 1

D. -2x + 5 ≥ 5

38. Given the graph of y = 1/3 x + 1 as shown, solve:



A. 1/3 x + 1 = 3

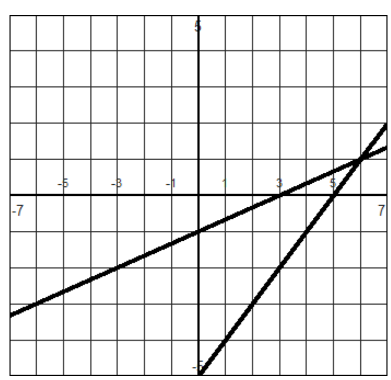
B. 1/3 x + 1 > 0

C. 1/3 x + 1 ≤ -1

39. Given the graph shown, solve:

A. 1/3x – 1 = x – 5

B. 1/3x – 1 > x – 5

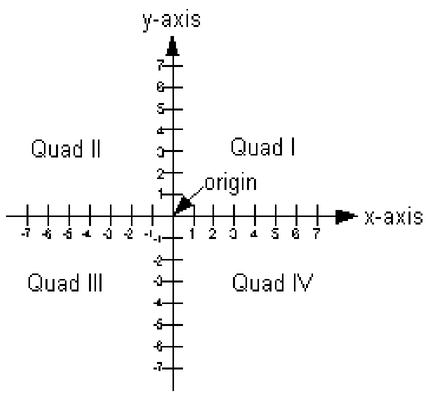


y=1/3x - 1

y=x-5

1.3 Graphs and Intercepts

The graph of an equation is a visual representation of the solutions to the equation. A point is included on the graph if its coordinates satisfy the equation. Points are plotted on the Cartesian (or rectangular) coordinate system. An example of which is shown.



The axes can be named appropriate to the problem situation but the horizontal axis represents the input variable and the vertical axis represents the output variable. The scale on the axes is determined by values appropriate to the individual problem and the scale on the axes may be different.

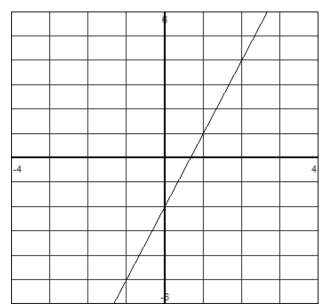
Solutions to equations are written as ordered pairs (x, y). To find a solution to an equation, we choose a value for one of the variables and solve for the other.

Example:

Fill in the table and use the table to construct a graph of the equation y = 3x – 2.

x	y
-1	**-5**
0	**-2**
1	**1**
2	**4**

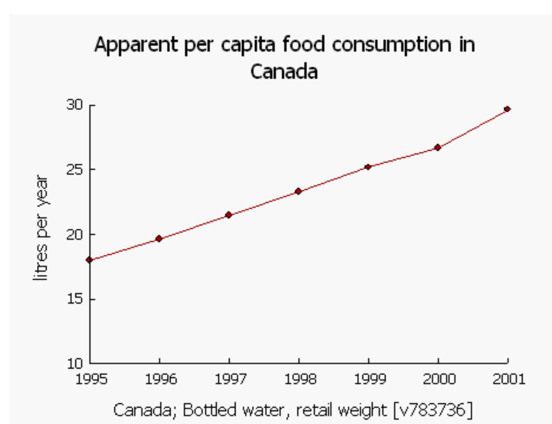
When x = -1, y = 3(-1) – 2 = -5
When x = 0, y = 3(0) – 2 = -2
When x = 1, y = 3(1) – 2 = 1
When x = 2, y = 3(2) – 2 = 4



Graphs can be used to show the relationship between two variables.

Example:

The graph shows the average amount of bottled water (in liters per year) consumed per person in Canada for the years from 1995 to 2001.



1. What is the average amount of bottled water consumed per person in 1996?

   There was approximately 20 liters per year consumed in 1996.

2. During what year was 25 liters per year consumed per person? 1999

A **complete graph** is a graph that shows all the key points and the behavior of the graph. Linear graphs should include the intercepts of the graph unless the intercept is not appropriate in the context of the problem.

The **intercepts** of a graph are the points where the graph intersects the axes. The horizontal intercept (or x-intercept) is the point where the graph crosses the x-axis. The vertical intercept (or y-intercept) is the point where the graph crosses the y-axis.

To find the intercepts:

1. The horizontal intercept is found by replacing the output variable (or y) with zero in the equation. Then solve for the remaining variable. You will have a point of the form (a, 0).

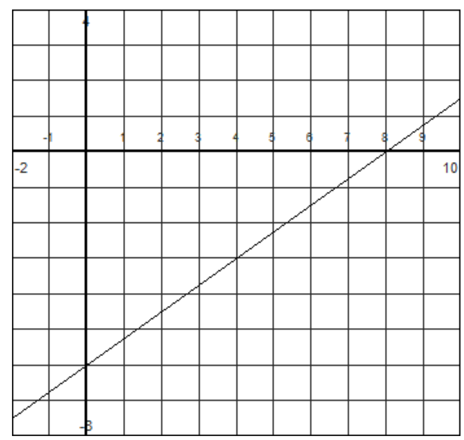
2. The vertical intercept is found by replacing the input variable (or x) with zero in the equation. Then solve for the remaining variable. You will have a point of the form (0, b).

Example:

1. Find the intercepts of the equation $3x - 4y = 24$.

   A. To find the x-intercept, set $y = 0$.
   $3x - 4(0) = 24$
   $3x = 24$
   $x = 8$ so the x-intercept (horizontal intercept) is the point (8,0).

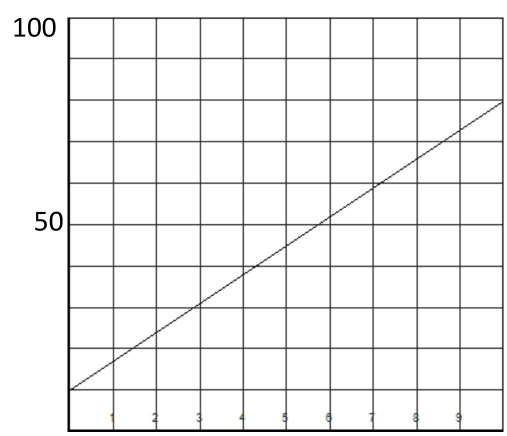
   B. To find the y-intercept, set $x = 0$.
   $3(0) - 4y = 24$
   $-4y = 24$
   $y = -6$ so the y-intercept (vertical intercept) is (0, -6).
   A complete graph is shown to the right.



2. To order specialized M&Ms the cost is given by $C = 9.99 + 6.99P$ where P is the number of pounds ordered and C is the total cost. Find the intercepts and discuss if they are reasonable in the context of the problem.

   A. To find the C-intercept, set $P = 0$.
   $C = 9.99 + 6.99(0) = 9.99$ so intercept is (0, 9.99)
   When 0 pounds of M&Ms are bought, the cost is $9.99.
   (This is the cost for shipping and handling).

   B. To find the P-intercept, set $C = 0$.
   $0 = 9.99 + 6.99P$
   $-6.99P = 9.99$
   $P = -1.43$ so intercept is (-1.43, 0)
   This would mean that if you bought a negative number of pounds of M&Ms then the cost would be $0. Negative pounds is not reasonable. This intercept would not need to be shown on the graph. A complete graph is shown.



Using the graphing calculator to make a table:

Step 1: Press $\boxed{Y =}$ Input your equation (in y = form).

Step 2: Go to the table setup (TBLSET) menu. Decide whether you want the calculator to input the values or whether you want to choose individual values. If you want the calculator to input the values, choose a value for TblStart (the starting value) and ΔTbl (what the values are increased by) and set Indpnt to Auto. If you want to choose individual values for the table, then set Indpnt to Ask.

Step 3: Go to TABLE.

Using the graphing calculator to make a graph:

Step 1: Press $\boxed{Y =}$ Input your equation (in y = form).

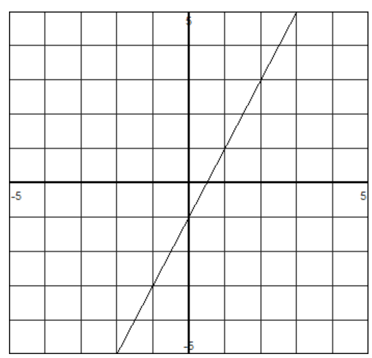
Step 2: Go to WINDOW. Enter an appropriate viewing window for the equation. This window should show a complete graph and the graph should be easy to read. Be sure to pick a scale for the axes that is appropriate to the values for the minimum and maximum values.

Step 3: Go to GRAPH.

<u>**Homework:**</u>

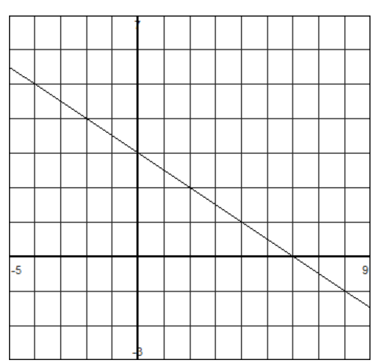
1. Use the graph to fill in the missing table values.
   Assume the scales are 1 unit.

x	y
0	
	3
-2	
	-3



2. Use the graph to fill in the missing table values.
   Assume the scales are 1 unit.

x	y
0	
	0
-2	
	-1



Fill in the table of values for the given equation.

3. 3x + 2y = 8

x	y
0	
	0
2	
	-2

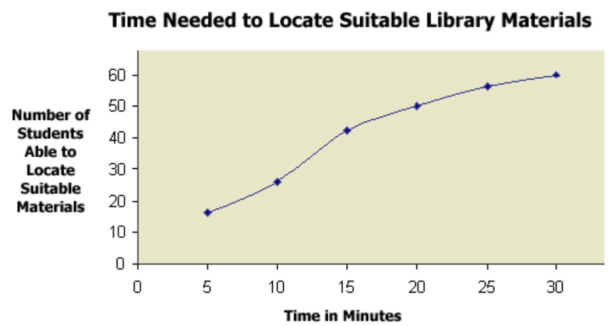
4. y = -2x + 1

x	y
0	
	0
4	
	-9

5. $\frac{1}{2}x - y = -4$

x	y
0	
	0
-4	
	8

6. Given the graph as shown, answer the following questions.

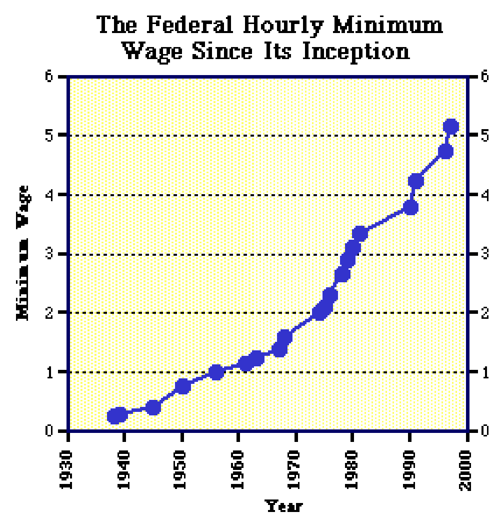


A. Approximately how many students can find suitable materials in 20 minutes?

B. What is the minimum time for at least 30 students to find suitable materials in the library?

C. Given at least 10 minutes, how many students could locate suitable materials?

7. Given the graph as shown, answer the following questions.



A. What was the approximate minimum wage in 1990?

B. What was the approximate minimum wage in 1980?

C. In what years did the minimum wage exceed $2?

D. When did the minimum wage increase the fastest?

E. Approximately how much did minimum wage rise during the 1960s?

8. Given 2x – 3y = 9:

A. Solve the equation for y in terms of x.

B. Find the horizontal intercept and vertical intercepts.

C. Sketch the graph.

Find the intercepts of the following equations. Then, find an appropriate viewing window including the scale on the axes.

9. 2L + 3W = 90

10. y = $\frac{3}{4}x + 6$

11. 0.50C + 1.75P = 350

12. 200x + 350y = 20

13. There is a relationship between the outside temperature and the rate of travel (speed) of certain ants. Several observations were made (which are shown in the table below) where s is the speed in centimeters per second and t is the temperature in degrees Celsius.

temperature	4	16	22	28
speed	0	2	3	4

A. What does the point (4, 0) represent in context?

B. Find an appropriate viewing window for the table values.

14. Given the equation: $y = \frac{2}{7}x - \frac{4}{5}$,

A. Fill in the table.

x	1	3	7	12	21
y					

B. Find an appropriate viewing window.

15. Cathy buys a 35-pound bag of rice and consumes 0.6 pounds per week.

A. Write an expression for the amount of rice that Cathy has left in terms of the number of weeks since she has bought the bag.

B. Find the amount of rice Cathy has left after 9 weeks.

C. Find the horizontal intercept and explain what it means in the context of this problem.

D. Find the vertical intercept and explain what it means in the context of this problem.

E. Sketch a complete graph of this equation.

16. The number of chirps a cricket makes per minute is given by N = 4T-160 where T is the temperature in degrees Fahrenheit.

A. How many chirps per minute does a cricket make when it is 90°F?

B. If a cricket makes 45 chirps per minute, what temperature is it?

C. Does the point (60, 80) satisfy the equation? If so, explain what this point represents in this problem situation.

D. Find the horizontal-intercept. Explain what this point represents in this problem situation.

E. Find the vertical intercept. Does this point make sense in the context of this problem? Why or why not?

F. Give a viewing window that would show a complete graph of this equation in context.

G. Sketch a complete graph of this equation.

17. Jan buys a car. The value of her car is given by the equation V = 25000-3500t, where t is the number of years since she bought the car.

A. Fill in the table.

t, years	0	1	3	4.5	6.25
V, value in dollars					

B. Find an appropriate viewing window for this problem scenario.

18. The temperature in the desert at 6 am was 65°F. The temperature rose 6 degrees every hour until it reached its maximum value at 4 pm.

A. Complete the table of values for the temperature, T, at h hours after 6 am.

hours, h	temperature, T
0	
5	
8	

B. Find an equation for the temperature, T, in terms of hours, h, since 6am.

C. When will the temperature be 90°F?

D. Sketch a graph for this problem situation. Be sure to use an appropriate viewing window.

1.4 Tables, Graphs, and Equations

Situations can be modeled mathematically using tables, graphs, and equations. A table of values displays specific data points, a graph is a visual representation that can be used to identify trends and overall behavior, and equations can be used to analyze and make predictions.

Examples:

1. Sarah buys a $50 phone card when she travels abroad. To call home, calls cost 2.8 cents per minute.

   A. Make a table of values showing the value of the card, V, for various lengths of time talked.

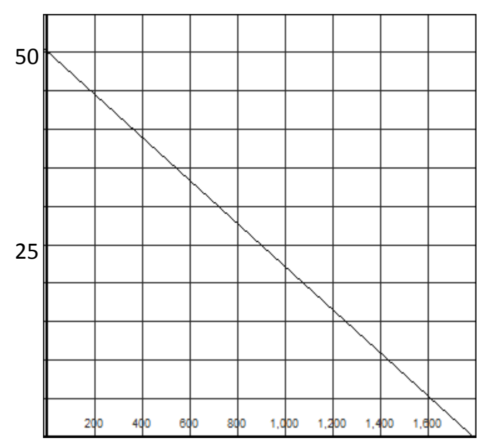
   The card starts at an amount of $50 which will decrease by 2.8 cents for every minute Sarah talks on the phone. So the value will be $50 minus $0.028 times the number of minutes talked.

Minutes talked, t		Value of card in dollars, V
0	50 – 0.028(0)	50
100	50 – 0.028(100)	47.20
500	50 – 0.028(500)	36
1000	50 – 0.028(1000)	22

   B. Plot the points on a graph and draw a line through the data points. Be sure to show the intercepts on the graph.

   Using the points found above and some additional points, the graph is shown here.

t	V
0	50
500	36
1000	22
1500	8
1750	1



   The vertical intercept is (0, 50) which represents the starting value of the card. The horizontal intercept is between 1750 and 1800 minutes. Negative values for time or the value of the card are not reasonable so the graph should be only shown in quadrant I.

   C. Write an equation for V in terms of t.

   Looking back to the calculations in part A, to find the value of the card:
   $V = 50 - 0.028t$, where t is in minutes and V is in dollars.

2. James is going to join a fitness club. The club has an initial fee of $79 and then a monthly fee of $39.99 per month.

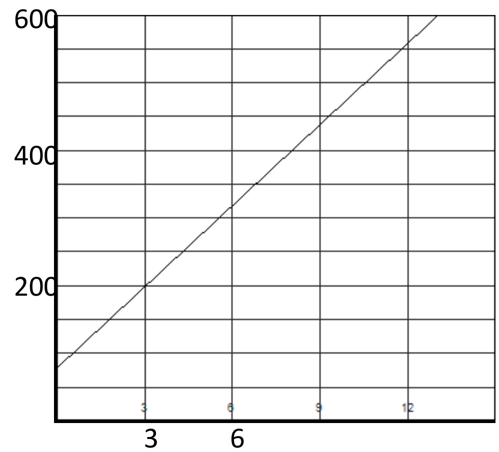
A. Make a table of values showing the cost of his membership in terms of the number of months he uses the club.

James has an initial fee of $79 and then additional fees of $39.99 each month he belongs to the club. So, his total fees will be the initial fee plus his monthly fees.

Months, M		Total fees, F
0	79 + 39.99(0)	79
3	79 + 39.99(3)	198.97
6	79 + 39.99(6)	318.94
12	79 + 39.99(12)	558.88

B. Plot the points on a graph and draw a line through the data points. Be sure to show the intercepts on the graph if reasonable.

The vertical intercept is at (0, 79) which represents the initial fee. The horizontal intercept would be at a negative number of months so it is not reasonable in the context of the problem. The graph should be only shown in quadrant I.



C. Write an equation for V in terms of t.

Looking at the calculations in part A, to find the total fee: F = 79 + 39.99M, where M is in months and F is in dollars.

Homework:

For problems 1-4, make a table of values with 4 entries, write an equation for the situation using appropriate variables, and make a complete graph. Find the intercepts and interpret them in context or discuss why the intercept is not reasonable.

1. Jennifer chooses a cell phone plan which has an activation fee of $35 and a monthly fee of $54.99. She must keep the plan for at least 2 years.

2. A deep-sea diver is taking reading as he rises from a depth of 350 feet. He is rising at a rate of 15 feet per minute. Since he starts at 350 feet below sea level his original depth is -350 feet.

3. Peggy won a 50 pound bag of pinto beans. Her family consumes about 0.75 pounds of pinto beans per week. Write an expression for the amount of pinto beans Peggy has left in terms of the number of weeks since she won the bag.

4. Chris makes $7.50 working at a local bookstore. He can only work between 0 and 30 hours per week.

5. A taxi fare in Philadelphia, PA is \$2.70 for the first 1/10 mile and then an additional \$0.23 per each 1/10 of a mile.

   A. Write an equation for this problem situation.

   B. If Greg takes a taxi 5 miles, what is his taxi fare?

   C. If Amanda has a taxi fare of \$10.98, what far did she travel?

   D. Philadelphia also has a flat rate of \$28.50 to travel anywhere within the city center. For which distances is the flat rate a better choice?

6. Barry works for a local furniture store. He makes a flat amount of \$250 per week plus 7% of his sales for the week.

   A. Write an equation for this problem situation.

   B. If Barry sells \$3700 worth of furniture, what is his pay for the week?

   C. If Barry needs to make \$600 per week, how much furniture does he need to sell?

7. Sharon is shopping for a new washing machine. She has narrowed her choices to two machines. One machine costs \$750 plus \$32 per year and the other more energy efficient machine costs \$900 plus \$18 per year.

   A. Write equations for the total cost of each washing machine in terms of the number of years she owns it.

   B. If she decides to buy the more energy efficient model, how long will it be before she starts saving money?

8. A money market account pays 1.75% annual interest on the amount deposited.

   A. Write an equation for the amount in the account after one year in terms of the amount deposited.

   B. If \$7500 is deposited, how much will be in the account at the end of the year?

   C. If a person wanted \$1000 in the account at the end of the year, how much would they need to deposit?

9. The boiling point of water changes with altitude. At sea level, water boils at 212°F, and the boiling point decreases by approximately 0.002°F for every 1-foot increase in altitude.

   A. Complete the table of values.

Altitude, a	-500	0	1000	2000	3000	4000	5000
Boiling Point, B							

   B. Write an equation for the boiling point, B, in terms of the altitude, a, in feet.

   C. What would be an appropriate window to view this data on your calculator?

   D. At what altitudes is the boiling point less than 204°F?

10. Jorge is pricing long distance phone service plans. He can choose from Plan A which costs \$0.20 a minute or Plan B which costs \$9.99 a month plus \$0.16 a minute. If he chooses Plan B, how many minutes will he have to talk before he starts saving money?

A. What are you asked to find in this problem? Assign a variable to represent it.

B. Write expressions in terms of your variable for the total cost incurred on each plan.

C. Write an equation to solve the problem. Solve the equation.

1.5 Functions and Function Operations

A function is a rule which assigns exactly one unique output value to each input value. Functions may be defined by a table, a graph, a formula, or verbally. **Note:** Different input values may have the same output value.

Numerically: Check that each unique input value in the table has exactly one output value associated with it.

Examples:

Determine whether or not each of the following tables represents a function.

1.

Age (years)	Height (inches)
5	40
9	45
13	60
17	66

For each age, there is one height associated with that age. This table represents a function.

2.

Time (seconds)	Height (feet)
1	3
2	51
3	67
4	51

This table represents a function. For each time there is exactly one height. An object can't be at two different heights at the same time.

3.

Federal Tax Rate	Income $
10%	0-8699
15%	8700-35349
25%	35350-85649
28%	85650-178649
33%	178650-388349
35%	388350+

This table does not represent a function because a tax rate of 10% has many possible income amounts. If one input has more than one output then it is not a function.

4.

x	y
0	6
-6	0
0	-6
6	0

This table does not represent a function because an input value of x = 0 results in two output values of 6 and -6.

Graphically:

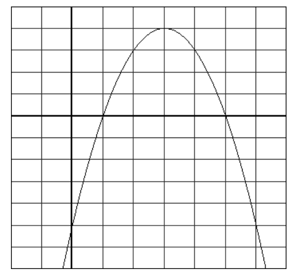
Vertical Line test: If a vertical line can be drawn that crosses the graph in more than one point, then the graph does not represent a function.

Note: All you have to find is one vertical line that touches the graph in more than one point and the graph does not represent a function.

Examples:

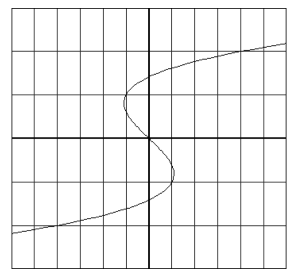
1. Is the graph shown the graph of a function?

   Yes, every vertical line that can be drawn will only cross the graph at one point. This graph passes the vertical line test.



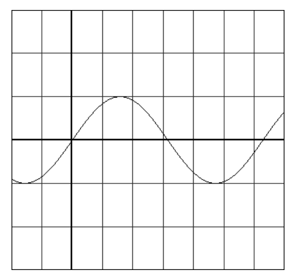
2. Is the graph shown the graph of a function?

   No, a vertical line drawn at x = 0 will cross the graph in three points (more than one point). This graph fails the vertical line test.



3. Is the graph shown the graph of a function?

   Yes, every vertical line that can be drawn will only cross the graph at one point. This graph passes the vertical line test.



Algebraically: For every input value, the equation should only produce one output value. To test if the equation represents a function, replace the input variable with a value, solve for the output value or values.

Examples:

1. Is $y = 2x^2 + x - 3$ a function?

   Yes, if you replace x with any value then there will be exactly one value of y. For example, when $x = 2$, $y = 2(2)^2 + 2 - 3 = 7$.

2. Is $x^2 + y^2 = 9$ a function?

   Replacing $x = 0$ yields $y^2 = 9$ and solving for y gives that y can be equal to both -3 and 3. Therefore, one input value corresponds to two output values and this equation does not represent a function.

Verbally: First determine which quantity represents the input and the output. Then, determine whether it is possible to have more than one output value for a single input value.

Examples:

1. Is a student's current class schedule a function of the student?

   Yes, a single student will only have one schedule of classes for this term.

2. If the variables were interchanged in the example above, would it still represent a function?

   No, more than one student might have the same schedule during a term. So, one schedule could correspond to more than one student.

3. Is the height of an object a function of time?

   Yes, an object cannot be at more than one height at the same time.

Function notation: If a relation between variables is a function, then the relation can be expressed using function notation. Function notation has a form of f(input)=output. The letter f (which can be any letter) is the name of the function. The notation gives a compact way to write the relationship between the input and output. To use function notation, replace the input variable with the given value and evaluate to find the output.

Examples:

1. Given the function $f(x) = 3x + 5$, find the following.

   A. $f(2)$

      To find $f(2)$, the value of x is replaced with 2 yielding $f(2) = 3(2) + 5 = 11$. Therefore, $f(2) = 11$.

   B. $f(5d)$

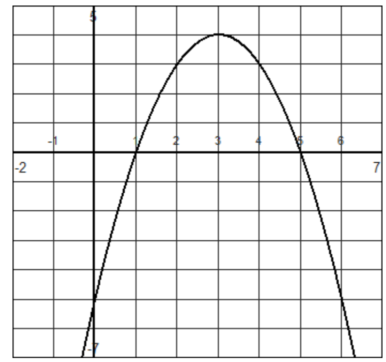
      $f(5d) = 3(5d) + 5$ so $f(5d) = 15d + 5$

2. The table shown gives the height of an individual at certain ages. Use the table to find $H(13)$.

   Looking at the table and finding where $A = 13$ gives $H(13) = 60$. This means that the individual was 60 inches tall at age 13.

A, years	H(A), inches
5	40
9	45
13	60
17	66

3. Given the graph of g(x) as shown to the right and assuming the scales on both axes are 1 unit, evaluate the following.



   A. g(1)

      Finding the point of the graph which has an x-value of 1, we find the y-value of that point. The graph contains the point (1, 0) so when x = 1 then g(1) = 0.

   B. g(3)

      Looking at where x = 3, the point on the graph is at (3, 4) so g(3) = 4.

Combining Functions

Functions can be combined to make new functions using mathematical operations. They may be added, subtracted, multiplied, divided and composed.

Operations on functions

If f(x) and g(x) both exist, then the sum, difference, product and quotient are defined as shown:

(f + g)(x) = f(x) + g(x)

(f – g)(x) = f(x) – g(x)

(fg)(x) = f(x) · g(x)

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, where g(x) ≠ 0

Examples:

1. Given f(x) = 3x – 7 and g(x) = 5x + 2, find (f + g)(x), (f – g)(x), (fg)(x) and $\left(\frac{f}{g}\right)(x)$.

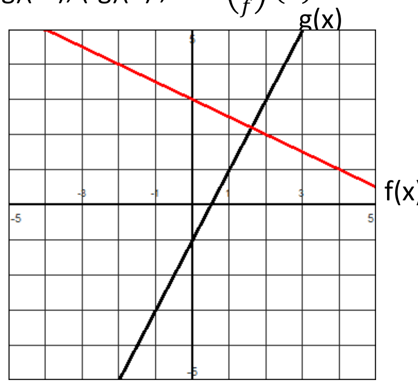
   (f + g)(x) = f(x) + g(x) = (3x – 7) + (5x + 2) = 8x – 5

   (f – g)(x) = f(x) – g(x) = (3x – 7) – (5x + 2) = -2x – 9

   (fg)(x) = f(x) · g(x) = (3x – 7)(5x + 2) = $15x^2$ – 29x – 14

   $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x-7}{5x+2}$

2. Given the graphs of f(x) and g(x) as shown, find (f + g)(-2), (fg)(3) , and $\left(\frac{g}{f}\right)(0)$.



(f + g)(-2) = f(-2) + g(-2) = 4 + -5 = -1

(fg)(3) = f(3)·g(3) = (1.5)(5) = 7.5

$\left(\frac{g}{f}\right)(0) = \frac{g(0)}{f(0)} = \frac{-1}{3}$

3. Given f(x) = x^2 – 3 , find f(a + h), f(a + h) – f(a), and $\frac{f(a+h)-f(a)}{h}$.

f(a + h) = $(a + h)^2 - 3 = a^2 + 2ah + h^2 - 3$

f(a + h) – f(a) = $[(a + h)^2 - 3]-[a^2 - 3] = [a^2 + 2ah + h^2 - 3] - [a^2 - 3] = 2ah + h^2$

$\frac{f(a+h)-f(a)}{h} = \frac{2ah+h^2}{h} = \frac{h(2a+h)}{h}$ = 2a + h

Composition of functions

If f and g are functions, then the composite function f ∘ g, or the composition of f and g is defined by

(f ∘ g)(x) = f(g(x)) and is read f of g of x. The domain of the composition of two functions consists of numbers which are in the domain of the inside function, g(x) and also in the domain of f(g(x)).

Examples:

1. Given f(x) = 2x – 5 and g(x) = x^2 – 3, find (f ∘ g)(x) and (g ∘ f)(x). Find the domain of each composite function.

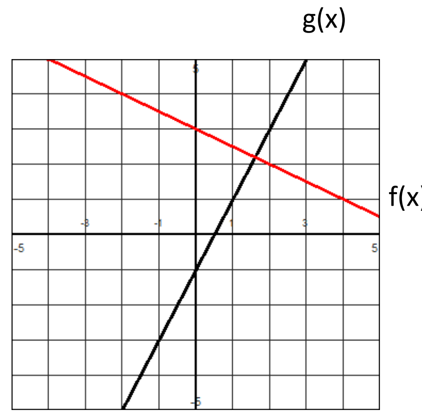
(f ∘ g)(x) = f(g(x)) = f(x^2 – 3) = 2(x^2 – 3) – 5 = 2x^2 – 11

Since both g(x) = x^2 – 3 and (f ∘ g)(x) = 2x^2 – 11 have a domain of all real numbers, then the domain of the composite function is all real numbers.

(g ∘ f)(x) = g(f(x)) = g(2x – 5) = $(2x - 5)^2$ – 3 = 4x^2 – 20x + 25 – 3 = 4x^2 – 20x + 22

Since both f(x) = 2x – 5 and (f ∘ g)(x) = 4x^2 – 20x + 22 have a domain of all real numbers, then the domain of the composite function is all real numbers.

2. Use the graphs shown to find $(f \circ g)(-1)$ and $(g \circ f)(4)$.



$(f \circ g)(-1) = f(g(-1)) = f(-3) = 4.5$

$(g \circ f)(4) = g(f(4)) = g(1) = 1$

3. Use the table shown to find, $(f - g)(3)$, $(fg)(1)$ and $(f \circ g)(0)$.

x	f(x)	g(x)
0	7	2
1	5	4
2	-1	0
3	3	1
4	0	9

$(f - g)(3) = f(3) - g(3) = 3 - 1 = 2$

$(fg)(1) = f(1) \cdot g(1) = (5)(4) = 20$

$(f \circ g)(0) = f(g(0)) = f(2) = -1$

4. Find the domain of $(f \circ g)(x)$ when $f(x) = \frac{1}{x-1}$ and $g(x) = \sqrt{x}$.

The domain of g(x) is $[0, \infty)$.

$(f \circ g)(x) = \frac{1}{\sqrt{x}-1}$ The domain will consist of numbers except for those which make the denominator equal to zero. The denominator will be equal to zero at x = 1. Therefore, the domain of the composite function is $[0, 1) \cup (1, \infty)$.

Decomposition of Functions

To decompose a function means to find the two functions that when composed will make up the given function. We need to find an f(x) and g(x) such that $(f \circ g)(x) = h(x)$, where h(x) is the given function. The concept is to look for an inside function, g(x), and an outside function, f(x). This is usually easy to do if there are parentheses in the function. The inside function will be inside the parentheses. In other functions, it may be more difficult to see the decomposition. Keep in mind that there may be more than one way to decompose a function. The overall purpose is to write a complicated function in terms of its simpler parts.

Examples:

1. Find f(x) and g(x) such that $(f \circ g)(x) = h(x)$ where $h(x) = (3x + 1)^4$.

Given $h(x) = (3x + 1)^4$, we can identify an inside function as the terms inside the parentheses which would give $g(x) = 3x + 1$. Then, since these terms are raised to a power, the outside function would be $f(x) = x^4$. When composed, these two functions will give $(3x + 1)^4$.

2. Find f(x) and g(x) such that (f ∘ g)(x) =h(x) where h(x) = $|x^2 - x + 2|$.

   Given h(x) = $|x^2 - x + 2|$, we can see that $x^2 - x + 2$ is inside the absolute value signs so this is what we would choose for g(x). The outside function would be the absolute value function. So, f(x) = $|x|$ and g(x) = $x^2 - x + 2$.

3. Find f(x) and g(x) such that (f ∘ g)(x) =h(x) where h(x) = 4^{x^3}.

   In this case, the exponent of the function would be chosen for the inside function and the exponential equation would be the outside function. Therefore, f(x) = 4^x and g(x) = x^3.

Homework:

1. Are the following functions?

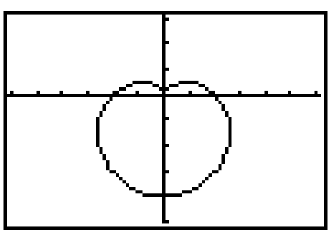
A.

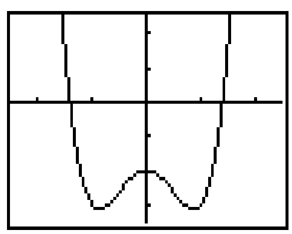
x	w
1	7
3	7
4	-1
8	3
-3	-1

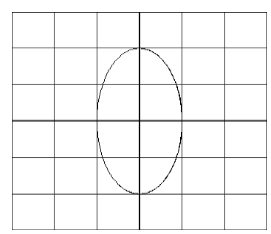
B.

t	h
-3	5
-2	6
-1	7
-2	8
-3	9

2. Are the following functions?

A. 

B. 

C. 

3. Is temperature in Celsius a function of temperature in Fahrenheit?

4. Is postage a function of weight of a letter?

5. Is starting salary a function of years of schooling?

6. Is 3x + 5y = 12 a function?

7. Is $x^2 + 2y^2 = 24$ a function?

8. Is x + $|y|$= 6 a function?

Evaluate the following functions.

9. Given $F(x) = 5x^3 - 8x + 1$, find

A. F(-2)

B. F(4)

C. $F(h^2)$

D. F(t) – 3

10. Given $F(t) = \dfrac{t+1}{2t-3}$, find:

A. F(2)

B. F(n + 1)

C. F(2g) + 1

D. 3F(a)

11. Given $g(t) = 3t^2 - 5t$, find:

A. g(-4)

B. g(3)

C. g(a) – 11

D. g(x + h)

12. Use the table shown to the right to find:

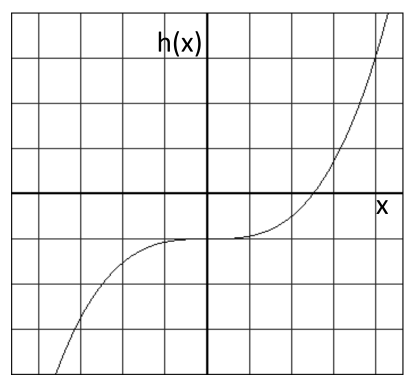
t	g(t)
2	11
3	-7
4	-1
8	5
10	9

A. g(4)

B. g(10)

C. For what t-value is g(t) = 11?

13. Assume the scales are 1 unit on both axes. Approximate the following function values from the graph.



A. h(0)

B. h(4)

C. h(-2)

D. For what x-value(s) is h(x) = -2?

14. Given f(x) = 6x – 1 and g(x) = x + 7, find:

A. (f + g)(x)

B. (f – g)(x)

C. (fg)(4)

D. $\left(\frac{f}{g}\right)(x)$

E. $(f \circ g)(x)$

F. $(g \circ f)(-2)$

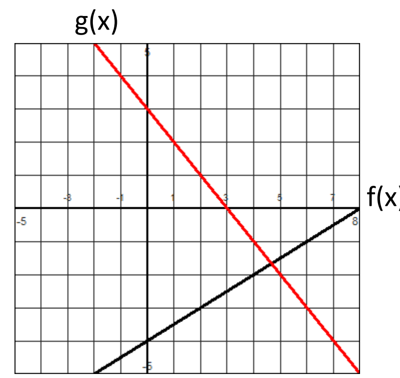
15. Given $f(x) = x^2$ and $g(x) = 2x + 7$, find:

A. $(f + g)(3)$ B. $(f - g)(x)$

C. $(fg)(x)$ D. $\left(\frac{f}{g}\right)(x)$

E. $(f \circ g)(1)$ F. $(g \circ f)(x)$

16. Given the graph shown, find the following:



A. $(f + g)(4)$ B. $(f - g)(1)$

C. $(fg)(-1)$ D. $\left(\frac{f}{g}\right)(1)$

E. $(f \circ g)(3)$

17. Given the table shown, find the following:

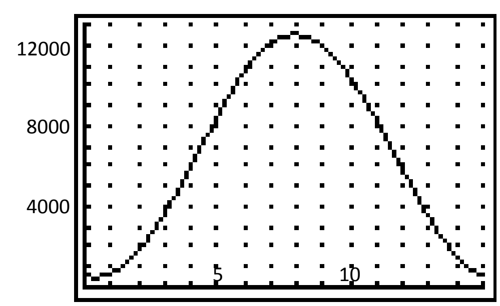
x	-1	0	1	2	3	4	5
f(x)	5	4	3	2	1	0	-1
g(x)	9	5	1	-3	-7	-11	-15

A. $(f + g)(3)$ B. $(g - f)(1)$

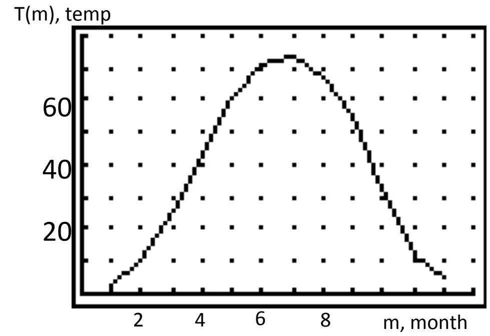
C. $(fg)(-1)$ D. $(g \circ f)(3)$

18. The graph shows S as a function of w. S represents the weekly sales of a best-selling book w weeks after it is released.

A. Find S(3) and explain what this represents in the context of the problem.
B. In which weeks were sales over $8000?
C. Approximately, what week had the highest sales? What were the sales that week?



19. The graph shows the graph of the average high temperatures in Fairbanks, Alaska for January (month 1) through December (month 12).



A. What month has the highest average temperature? Approximately what is the temperature during that month? Write this point in function notation.

B. Estimate T(4) and explain what this means in context.

C. During which months is the average high temperature below 30 degrees?

For 20-25, find f(x) and g(x) so that h(x) = $(f \circ g)(x)$.

20. h(x) = $\sqrt{3x^2+4}$

21. h(x) = $(x^2 + x + 1)^5$

22. h(x) = $|2x - 1|$

23. h(x) = 5^{3x+1}

24. h(x) = $\dfrac{1}{(2x+3)^2}$

25. h(x) = $\dfrac{x^2+1}{x^2}$

26. For f(x) = 4x + 7, find:

A. f(a +h) B. f(a+h) – f(a) C. $\frac{f(a+h)-f(a)}{h}$

27. For f(x) = x^2 + x, find:

A. f(a +h) B. f(a+h) – f(a) C. $\frac{f(a+h)-f(a)}{h}$

28. For f(x) = $\frac{1}{x}$, find:

A. f(a +h) B. f(a+h) – f(a) C. $\frac{f(a+h)-f(a)}{h}$

29. For f(x) = x^3 – 1, find:

A. f(a +h) B. f(a+h) – f(a) C. $\frac{f(a+h)-f(a)}{h}$

30. Find the domain of (f ∘ g)(x) when f(x) = $\frac{1}{x-2}$ and g(x) = $\frac{1}{x}$.

31. Find the domain of (f ∘ g)(x) when f(x) = x^3 + 3 and g(x) = $\sqrt{x}$.